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THE EFFECT OF DIFFUSION RATE ON THE FLOW OF GAS MIXTURES

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The laws of motion and heat exchange of gas mixtures are considerably more complex than of the one-component gases. It is ovbious on physical grounds that the determining parameters (density, velocity and temperature) have generally different mean values for the various components. This results in the appearance of other macroscopic parameters of flow, in particular, of the mean diffusion rate.

The presence of such nonzero macroscopic flow parameters violates the fundamental laws of conservation for components of gas. For each of these only the law of conservation of mass remains valid, while the laws of conservation of momentum and energy are no longer satisfied. The motion of individual components is not defined by equations of conventional aerodynamics such as the Euler, the Navier-Stokes or Barnett's equations. A redistribution of temperature and flow velocities takes place between individual components of gas. Moreover, as will be shown subsequently, diffusion rates generate internal stresses and thermal fluxes which tend to increase the entropy of the gas mixture and to accelerate its approach to equilibrium.

These additional macroscopic flow parameters substantially affect the motion and heat exchange in the gas mixture and are, thus, fundamental parameters of the system. In spite of this, they are not determined sufficiently accurately. The classical Chapman-Enskog method is used for solving the system of kinetic equations on the assumption that diffusion rates and temperatures vanish in the zero approximation [1]. In that method the diffusion rate appears only in the first approximation and is naturally inaccurately determined.

As recently shown, stresses and heat fluxes make their appearance in the second approximation thogether with a whole group of additional Barnett terms. Several attempts at refining the theory of multicomponent gas mixtures have been published. The work of Kolodner [2] should be noted in which Grad's method was applied to a gas mixture and a number of complicated integrals were computed, and, also, the work of Sirovich [3] in which Kruka's model equation is used in the case of two-component gas mixtures and the method of iteration is substituted for that of successive approximations.

Several variants of the method of successive approximations — the method of small parameter — is used here for solving the system of kinetic equations of gas mixtures. This is made possible by the presence of the previously mentioned macroscopic flow parameters which permits the use of various methods of solution construction.

The system of kinematic equations can be written in the form

$$\frac{\partial f_s}{\partial t} + \mathrm{v}\frac{\partial f_s}{\partial \mathbf{r}} = \sum_{\tau=1}^{M} \iiint (f_s' f_\tau' - f_s f_\tau)\, q_{s\tau} b\, db\, d\varepsilon \tag{1}$$

The density, mean velocity and mean temperature of the s-th component of gas are uniquely defined by the distribution function $f_s(\mathbf{r}, \mathrm{v}, t)$, while the distribution function

$$f(\mathbf{r}, \mathrm{v}, t) = \sum_{s=1}^{M} f_s(\mathbf{r}, \mathrm{v}, t)$$

makes it possible to determine these parameters for the flow of the gas mixture as a whole.

The first method. The system of kinematic equations is solved by the classical Chapman-Enskog method described in the monograph [1]. Let us consider this method in detail in order to elucidate its peculiarities, and some of its advantages and shortcomings.

It is assumed in this method that the probability of collisions between gas molecules of one and the same kind and those of different kinds is the same, which presupposes that the gas mixture is fairly uniform. Hence in the first approximation diffusion rates and temperatures are zero. Since all integrals of collisions in the right-hand sides of the system of Eqs. (1) are of the same order, that system can be presented in the form

$$\frac{df_s}{dt} = \frac{1}{\varepsilon} \sum_{\tau=1}^{M} J(f_s, f_\tau) \tag{2}$$

where ε is a small parameter. Let us seek a solution of the form

$$f_s = f_s^{(0)} + \varepsilon f_s^{(1)} + \varepsilon^2 f_s^{(2)} + \ldots \tag{3}$$

In the zero approximation we have

$$f_s^{(0)} = n_s \left(\frac{m_s}{2\pi kT}\right)^{3/2} \exp\left[-\frac{m_s(\mathrm{v} - \mathrm{U})^2}{2kT}\right]$$

where n_s is the density of the s-th component of gas. The mean velocity u_s and temperature T_s of the s-th component in the zero approximation are equal to the mean velocity U and temperature T of the whole stream. The diffusion rates W^s in that approximation are naturally zero. The system of kinetic equations yields the following system of transport equations [1]:

$$\frac{\partial n}{\partial t} + \frac{\partial n U_\alpha}{\partial r_\alpha} = 0, \qquad \frac{\partial U_\alpha}{\partial t} + U_\beta \frac{\partial U_\alpha}{\partial r_\beta} = -\frac{\partial P_{\alpha\beta}}{\partial r_\beta}$$

$$\frac{{}^3/_2\, \partial p}{\partial t} + \frac{{}^3/_2\, \partial p U_\alpha}{\partial r_\alpha} = -P_{\alpha\beta} \frac{\partial U_\alpha}{\partial r_\beta} - \frac{\partial q_\alpha}{\partial r_\alpha}$$

where $P_{\alpha\beta}$, q_α and W_α^s are determined by the known expressions in terms of the distribution function.

In the zero approximation we have

$$P_{\alpha\beta} = p\delta_{\alpha\beta}, \qquad q_\alpha = 0, \qquad W_\alpha^s = 0$$

In this approximation the motion of the gas mixture as a whole is defined by Euler's equations. The motion of individual components of gas does not affect in any way the over-all motion of gas.

In the first approximation we have

$$P_{\alpha\beta} = p\delta_{\alpha} - \sigma_{\alpha\beta}, \quad q_{\alpha} = -\lambda \frac{\partial T}{\partial r_{\alpha}}, \quad W_{\alpha}^{s} \neq 0$$

In this approximation for the diffusion rates in the stream we obtain the following expression:

$$\mathbf{W}^{s} = \frac{n^{2}}{n_{s}\rho} \sum m_{\tau} D_{s\tau} \mathbf{d}_{\tau} - \frac{1}{n_{s} m_{s}} D_{s}^{\tau} \operatorname{grad} \ln T$$

$$D_{s\tau} = \frac{\rho n_{s}}{2 n m_{\tau}} \sqrt{\frac{2kT}{m_{s}}}\, C_{s0}^{\tau s}(\xi), \qquad D_{s}^{T} = \frac{m_{s} n_{s}}{2} \sqrt{\frac{2kT}{m_{s}}}\, Q_{s0}(\xi)$$

where ξ is the number of terms of the expansion in Sonin's polynomials and vector $\mathbf{d}_{\tau}$ is defined by

$$\mathbf{d}_{\tau} = \operatorname{grad}\left(\frac{n_{\tau}}{n}\right) + \left(\frac{n_{\tau}}{n} - \frac{n_{\tau} m_{\tau}}{\rho}\right) \operatorname{grad} \ln p$$

When $\xi = 1$, $C_{s0}^{\tau s} \neq 0$ and $Q_{s0} = 0$. Coefficients $C_{s0}^{\tau s}$ are determined by a system of algebraic equations [1]. Thus in this approximation the motion of a gas mixture is defined by the Navier-Stokes equations, although the motion of its individual components does not explicitly affect the over-all motion of the gas mixture. For the determination of diffusion rates in the stream we obtain the following system of equations:

$$\sum_{\tau} \frac{n_{s} n_{\tau}}{n^{2} D_{s\tau}(1)} (\mathbf{W}^{\tau} - \mathbf{W}^{s}) = \mathbf{d}_{s}$$

where $D_{s\tau}$ is the coefficient of binary diffusion.

In the second approximation we have

$$P_{\alpha\beta} = p\delta_{\alpha\beta} - \sigma_{\alpha\beta} - \sum_{s=1}^{M} \rho_{s} W_{\alpha}^{s} W_{\beta}^{s}, \quad q_{\alpha} \neq 0, \quad W_{\alpha}^{s} \neq 0$$

It will be seen that in this approximation an additional stress tensor, which depends on the diffusion rate in the stream, appears beside the usual viscous tension tensor. The heat flux vector has also changed in this approximation.

Thus the diffusion rates affect directly the laws of motion and heat transfer of a gas mixture only in the second approximation. The most general form of the second approximation of the Chapman-Enskog method was obtained in [4], as suggested by the author.

The second method. We shall now assume that the probability of collision between molecules of the same kind is higher than that of collision between molecules of different gases, which is equivalent to the assumption of incomplete mixing of gas components in the mixture. Hence, even in the zero approximation the diffusion rates are not zero. In this case the collision integrals in the right-hand sides of system (1) are not of the same order, and the system of equations can be written in the form

$$\frac{df_{s}}{dt} = \frac{1}{\varepsilon} J(f_{s}, f_{s}) + \sum_{\tau \neq s}^{M} J(f_{s}, f_{\tau}) \tag{4}$$

We seek a solution in the form of series (3) and, using the method described in [5], set

$$f_{s} = f_{s}(\tau, \mathbf{v}, \mathbf{r}, n_{s}(\mathbf{r}, \tau), u_{s}(\mathbf{r}, \tau), T_{s}(\mathbf{r}, \tau)) \tag{5}$$

We represent the dependence of parameters n_{s}, u_{s} and T_{s} on fast time in accordance with [5] in the form

$$\frac{\partial n_s}{\partial t} = \varepsilon A_s^{(0)} + \varepsilon^2 A_s^{(1)} + \ldots \tag{6}$$

$$\frac{\partial u_s^\alpha}{\partial t} = \varepsilon B_{s\alpha}^{(0)} + \varepsilon^2 B_{s\alpha}^{(1)} + \ldots$$

$$\frac{\partial T_s}{\partial \tau} = \varepsilon C_s^{(0)} + \varepsilon^2 C_s^{(1)} + \ldots$$

where $A_s^{(k)}$, $B_{s\alpha}^{(k)}$ and $C_s^{(k)}$ are unknown functions which are to be determined.

Substituting (3) and (6) into (4), taking into account (5), and collecting terms of same powers, we obtain a recurrent system of equations for the determination of $f_s^{(k)}$ $(\mathbf{r}, \mathbf{v}, t)$ which for $\tau \gg \tau_1$ may according to [6] be written as

$$0 = J\left(f_s^{(0)}, f_s^{(0)}\right) \tag{7}$$

$$\left(A_s^{(0)} \frac{\partial}{\partial n_s} + B_s \frac{\partial}{\partial u_s^\alpha} + C_s^{(0)} \frac{\partial}{\partial T_s}\right) f_s^{(0)} + v_s{}^\alpha \frac{\partial f_s^{(0)}}{\partial r_\alpha} - \sum_{\tau \neq s} J\left(f_s^{(0)}, f_\tau^{(0)}\right) = J^*\left(f_s^{(1)}, f_s^{(0)}\right)$$

$$\left(A_s^{(0)} \frac{\partial}{\partial n_s} + \ldots\right) f_s^{(1)} + \left(A_s^{(1)} \frac{\partial}{\partial n_s} + \ldots\right) f_s^{(0)} + v_s{}^\alpha \frac{\partial f_s^{(1)}}{\partial r_\alpha} - \sum_{\tau \neq s} J^*\left(f_s^{(0)}, f_\tau^{(1)}\right) = J^*\left(f_s^{(2)}, f_s^{(0)}\right)$$

The first of Eqs. (7) has the following unique nontrivial solution:

$$f_s^{(0)} = n_s \left(\frac{m_s}{2\pi k T_s}\right)^{3/2} \exp\left[-\frac{m_s (\mathbf{v} - \mathbf{u}_s)^2}{2kT_s}\right]$$

where n_s, u_s and T_s are so far arbitrary functions r and t which define the s-th kind of gas. The solvability of consequent integral equations of system (7) requires the fulfilment of the following conditions:

$$\iiint \psi_s D_s^{(k)}(f_s)\, d\mathbf{v} = 0$$

where $\psi = m_s$, $m_s v$ and $m_s v^2$ are eigenfunctions of the integral operator of the right-hand part of (7), and $D_s^{(k)}(f_s)$ is the nonhomogeneous part of the integral equation in the k-th approximation. From these conditions we find all unknown functions

$$A_s^{(0)} = -\frac{\partial n_s u_s^\alpha}{\partial r_\alpha}; \qquad A_s^{(k)} = 0, \qquad k = 1, 2 \ldots$$

$$B_{s\alpha}^{(0)} = -u_s{}^\beta \frac{\partial u_s^\alpha}{\partial r_\beta} - \frac{1}{\rho_s} \frac{\partial P_s}{\partial r_\alpha} + \frac{1}{\rho_s} \sum_{\tau \neq s} B_{s\tau}^\alpha(0), \qquad B_{s\alpha}^{(k)} = -\frac{1}{\rho_s} \frac{\partial P_{\alpha\beta}^s(k)}{\partial r_\beta}$$

$$C_s^{(0)} = -u_s{}^\alpha \frac{\partial T_s}{\partial r_\alpha} - \frac{2}{3} T_s \frac{\partial u_s^\alpha}{\partial r_\alpha} + \frac{1}{3kn_s} \sum_{\tau \neq s} C_{\tau s}^{(k)}(0);$$

$$C_s^{(k)} = -\frac{1}{3kn_s} \frac{\partial Q_\alpha^s(k)}{\partial r_\alpha} - \frac{2P_{\alpha\beta}^s}{3kn_s} \frac{\partial u_s^\alpha}{\partial r_\beta}$$

where

$$P_{\alpha\beta}^s(k) = \iiint_{-\infty}^{+\infty} m_s (\mathbf{v} - \mathbf{u}_s)^\alpha (\mathbf{v} - \mathbf{u}_s)^\beta f_s^{(k)}\, d\mathbf{v}, \qquad Q_\alpha{}^s(k) = \iiint_{-\infty}^{\infty} 2m_s (\mathbf{v} - \mathbf{u}_s)^2 f_s{}^{(k)}\, d\mathbf{v}$$

and the notation

$$B^{\alpha}_{s\tau}(0) = \iiint I\left(f_s^{(0)}, f_\tau^{(0)}\right) m_s v^\alpha \, d\mathbf{v} = \frac{16}{3} \frac{\rho_s \rho_\tau}{m_s + m_\tau} \Omega^{(1,1)}_{s\tau} (\mathbf{u}_\tau - \mathbf{u}_s)^\alpha$$

$$C_{s\tau}(0) = \iiint I\left(f_s^{(0)}, f_\tau^{(0)}\right) m_s (\mathbf{v} - \mathbf{u}_s)^2 \, d\mathbf{v} = \frac{32}{3} \frac{\rho_s \rho_\tau}{m_s + m_\tau} \Omega^{(1,1)}_{s\tau} \frac{T_s}{m_\tau} \left(\frac{T_\tau}{m_\tau} + \frac{T_s}{m_s}\right)^{-1} \times$$
$$\left[\frac{3}{2} \frac{m_s}{m_s + m_\tau} \left(\frac{T_\tau}{T_s} - 1\right)\left(\frac{2kT_s}{m_s} + \frac{2kT_\tau}{m_\tau}\right) + (\mathbf{u}_\tau - \mathbf{u}_s)^2\right]$$

$$\Omega^{(l,n)}_{s\tau} = \sqrt{2\pi k \left(\frac{T_s}{m_s} + \frac{T_\tau}{m_\tau}\right)} \int_0^\infty \int_0^\infty e^{-q^2_{s\tau}} q^{2n+3}_{s\tau} (1 - \cos^l \chi) \, dq_{s\tau} b\, db$$

where $q_{s\tau}$ is the reduced initial velocity, χ is the angle of deflection, and b is the impact distance, is used. For $T_s = T_\tau$ these integrals coincide with the integrals presented in the monograph [1].

Hence it is necessary for the solvability of first approximation equations (7) that the gasdynamic parameters of gas satisfy the following system of equations:

$$\frac{\partial n_s}{\partial t} + \frac{\partial n_s u_s^\alpha}{\partial r_\alpha} = 0 \tag{8}$$

$$\frac{\partial u_s^\alpha}{\partial t} + u_s^\beta \frac{\partial u_s^\alpha}{\partial r_\beta} + \frac{1}{\rho_s} \frac{\partial p_s}{\partial r_\alpha} = \frac{16}{3\rho_s} \sum_{\tau \neq s}^{M} \frac{\rho_s \rho_\tau \Omega^{(1,1)}_{s\tau}}{m_s + m_\tau} (\mathbf{u}_\tau - \mathbf{u}_s)^\alpha$$

$$\frac{\partial T_s}{\partial t} + u_s^\alpha \frac{\partial T_s}{\partial r_\alpha} + \frac{2}{3} T_s \frac{\partial u_s^\alpha}{\partial r_\alpha} = \frac{16}{3\rho_s} \sum_{\tau \neq s} \frac{\rho_s \rho_\tau}{m_s + m_\tau} \Omega^{(1,1)}_{s\tau} \frac{T_s}{m_s} \left(\frac{T_s}{m_s} + \frac{T_\tau}{m_\tau}\right)$$
$$\left[\frac{3}{2} \cdot \frac{m_s}{m_s + m_\tau} \left(\frac{T_\tau}{T_s} - 1\right)\left(\frac{2kT_s}{m_s} + \frac{2kT_\tau}{m_\tau}\right) + (\mathbf{u}_\tau - \mathbf{u}_s)^2\right]$$

The obtained system of equations, similar to Euler's, shows that averaged stream parameters n_s, u_s and T_s, can be determined for each component of gas already in the zero approximation, thus making it possible to determine diffusion rates and temperatures. It will be seen that the equations of conservation of mass are strictly satisfied, while the equations of conservation of momentum and the equation of energy have now right-hand sides which for $W^s \neq W^\tau$ and $T_s \neq T_\tau$ are nonzero.

These right-hand parts determine the interaction between the various components of gas which tend to equalize the stream temperatures and velocities. The structure of these expressions, which were strictly derived for gas mixtures in the range of molecule mass ratio from unity to 200-300, is quite important. It is, however, reasonable to assume that these expressions can be used for determining with a good approximation finely dispersed media in a considerably wider range of variation of particle masses.

Equations (8) obtained by the new method readily yield the zero approximation relationship between diffusion rates which were derived by the Chapman-Enskog method in the first approximation. In fact, if we assume in addition that

$$\frac{\partial w^s}{\partial t} \approx 0, \qquad w_\beta{}^s \frac{\partial w^s_\alpha}{\partial r_\beta} \approx 0, \qquad T_s \approx T$$

and that parameters of the whole stream satisfy Euler's equations, the second of Eqs. (8)

yields

$$\sum_{\tau} \frac{n_s n_\tau}{n^\alpha D_{s\tau}(1)} (W^\tau - W^s) = d_s , \quad D_{s\tau} = \frac{3\,(m_s + m_\tau)}{16 \quad n m_s m_\tau} \frac{kT}{\Omega_{s\tau}^{(1,1)}} \tag{9}$$

where $D_{s\tau}$ is the coefficient of binary diffusion in the first approximation. It will be seen that expression (9) coincides with the one derived above.

The effectiveness of the new method is now evident, and it is clear that formulas for diffusion rates and the diffusion coefficients calculated by the Chapman-Enskog method are only very approximate. On the same assumptions from the last of Eqs. (8) we can obtain the expressions

$$\sum_{\tau} \frac{n_s n_\tau}{n^\alpha D_{s\tau}} \left[\frac{3km_s}{m_s + m_\tau} \left(\frac{T_s}{T_\tau} - 1 \right) \left(\frac{T_s}{m_s} + \frac{T_\tau}{m_\tau} \right) + \right.$$
$$\left. (\mathbf{u}_\tau - \mathbf{u}_s)^2 \right] \frac{T_s}{m_s} \left(\frac{T_\tau}{m_s} + \frac{T_\tau}{m_\tau} \right) = 0$$

which make it possible to determine the temperature difference between various components of gas by the diffusion rates in the stream.

The system of Eqs. (8) obtained in the zero approximation can, thus, be used for investigating gasdynamic flows and heat exchange phenomena in multicomponent gas mixtures. A similar system may serve as a reasonable approximation in the investigation of motion of finely dispersed systems.

To determine parameters of the stream in the first approximation we substitute into the second of Eqs. (7) the relationship between these parameters in the zero approximation by using formulas (8). After some simple transformations we obtain

$$f_s^{(0)} \left\{ 2\,(\omega_s \omega_s)^{\alpha\beta} \frac{\partial u_s^\alpha}{\partial r_\beta} + (\mathbf{v} - \mathbf{u}_s)^\alpha \left(\mathrm{W}_s^2 - \frac{5}{2} \right) \frac{\partial \ln T_s}{\partial r_\alpha} + \frac{(\mathbf{v} - \mathbf{u}_s)^2}{\rho_s} \sum_{\tau \neq s} B_{s\tau}^\alpha (0) + \right. \tag{10}$$
$$\left. \left(\omega_s^2 - \frac{3}{2} \right) \frac{1}{3p_s} \sum_{\tau \neq s} C_{s\tau}(0) \right\} - \sum_{\tau \neq s} J\,(f_s^{(0)}, f_s^{(0)}) = J^*\,(f_s^{(1)}, f_s^{(0)})$$
$$\omega_s = (\mathbf{v} - \mathbf{u}_s) \left(\frac{m}{2kT_s} \right)^{-1/2}$$

The sum of integrals in the left-hand part of this expression can be expanded in terms of irreducible tensors. Let us represent it in the form of expansion

$$\sum_{\tau \neq s} J_{s\tau}(0) = f_s^{(0)} \{ L_0 + (\mathbf{v} - \mathbf{u}_s)^\alpha L_{(1)}^\alpha + (2\omega_s^\alpha \omega_s^\beta - \sigma_{\alpha\beta})\, L_{(2)}^{\alpha\beta} + \tag{11}$$
$$\omega_s^\alpha (\omega_s^2 - {}^5\!/_2) L_{(3)}^\alpha + \ldots \}$$

Multiplication of each term of the right- and left-hand sides of this equation by $1, (v - u_s)^\alpha$, $(2\omega_s^\alpha \omega_s^\beta - \delta_{\alpha\beta})$ and $\omega_s^\alpha (\omega_s^2 - {}^5\!/_2)$ and integration yields

$$L_0 = 0,\ L_{(1)}^\alpha = \frac{1}{\rho_s} \sum_{\tau \neq s} B_{s\tau}^\alpha (0), \qquad L_{(3)}^\alpha = \frac{1}{\rho_s} \sum_{\tau \neq s} H_{s\tau}^\alpha$$
$$L_{(2)}^{\alpha\beta} = \frac{1}{2n_s} \sum_{\tau \neq s} \iiint (2\omega_s^\alpha \omega_s^\beta - \delta_{\alpha\beta})\, J_{s\tau}(0)\, dv$$

with the relationship

$$(2\omega_s^\alpha \omega_s^\beta - \delta_{\alpha\beta})\, L_{(2)}^{\alpha\beta} = 2 \frac{(\boldsymbol{\omega}_s \boldsymbol{\omega}_s)^{\alpha\beta}}{\rho_s} \sum_{\tau \neq s} D_{\tau s}^{\alpha\beta} + \left(\boldsymbol{\omega}_s^2 - \frac{3}{2} \right) \frac{1}{3p_s} \sum_{\tau \neq s} C_{s\tau}(0)$$

Susstituting (11) into formula (10), we obtain

$$f_s^{(0)}\left[(\omega_s\omega_s)^{\alpha\beta}\left\{2\frac{\partial u_s^{\alpha}}{\partial r_{\beta}}-\frac{1}{\rho_s}\sum_{\tau\neq s}D_{s\tau}^{\alpha\beta}\right\}+\right. \tag{12}$$
$$\left.\omega_s{}^{\alpha}\left(\omega_s{}^2-\frac{5}{2}\right)\left\{\left(\frac{2kT_s}{m_s}\right)^{1/2}\frac{\partial\ln T_s}{\partial r_{\alpha}}-\frac{1}{\rho_s}\sum_{\tau\neq s}H_{s\tau}^{\alpha}\right\}\right]=J^*\left(f_s^{(0)},f_s^{(0)}\right)$$

where in the particular case of Maxwellian molecules $D_{s\tau}{}^{\alpha\beta}$ and $H_{s\tau}{}^{\alpha}$ are defined by

$$D_{s\tau}^{\alpha\beta}=\iiint m_s(\omega_s\omega_s)^{\alpha\beta}J_{s\tau}(0)\,d\mathbf{v}=\left(\frac{m_sm_\tau}{m_s+m_\tau}\right)^2\frac{16n_sn_\tau}{6kT_s}\Omega_{s\tau}^{(11)}((\mathbf{u}_\tau-\mathbf{u}_s)(\mathbf{u}_\tau-\mathbf{u}_s))^{\alpha\beta}$$

$$H_{s\tau}^{\alpha}=\frac{8}{10}\iiint m_s\omega_s{}^{\alpha}\left(\omega_s{}^2-\frac{5}{2}\right)J_{s\tau}(0)\,d\mathbf{v}=\frac{8}{10}\frac{m_\tau m_s{}^2}{m_s+m_\tau}\frac{16}{3}n_\tau n_s\Omega_{s\tau}^{(11)}\times \tag{13}$$
$$\left\{\frac{5}{2}\left(\frac{T_\tau}{m_\tau}\left(\frac{T_\tau}{m_\tau}+\frac{T_s}{m_s}\right)^{-1}+1\right)\sqrt{\frac{m_s}{2kT_s}}(u_s-u_\tau)^{\alpha}-\left[2\frac{m_s{}^2}{(m_s+m_\tau)^2}\times\right.\right.$$
$$\left.\left.\left(1-\frac{T_s}{T_\tau}\right)^2+\frac{T_s{}^2}{T_\tau{}^2}\right]\right\}\left\{\left(\frac{T_\tau}{m_\tau}+\frac{T_s}{m_s}\right)^{-1}\frac{T_\tau}{m_\tau}\frac{T_\tau}{T_s}\sqrt{\frac{m_s}{2kT_s}}(\mathbf{u}_s-\mathbf{u}_\tau)^{\alpha}\right\}$$

It will be seen that expressions (12) represent the well known nonhomogeneous linear equations of the kinetic theory. We seek a solution of these integral equations of the form

$$f_s^{(1)}=f_s^{(0)}\left\{(\omega_s\omega_s)^{\alpha\beta}\left(\frac{\partial u_s^{\alpha}}{\partial r_{\beta}}-\frac{1}{2\rho_s}\sum_{\tau\neq s}D_{s\tau}^{\alpha\beta}\right)B_s+\omega_s{}^{\alpha}\left(\omega_s{}^2-\frac{5}{2}\right)\times\right.$$
$$\left.\left[\frac{\partial\ln T_s}{\partial r_s}\sqrt{\frac{2kT_s}{m_s}}-\frac{1}{\rho_s}\sum_{\tau\neq s}H_{\tau s}^{\alpha}\right]C_s\left(\frac{m_s}{2kT_s}\right)^{1/2}\right\}$$

This solution has been fully investigated in the case of a simple gas with B_s and C_s defined by

$$B_s=-\frac{5}{4n_s\,\Omega_{ss}^{(2,2)}},\qquad C_s=-\frac{15}{16}\sqrt{\frac{2kT_s}{m_s}}\frac{1}{n_s\,\Omega_{ss}^{(2,2)}} \tag{14}$$

Using the first approximation distribution function and (13) and (14), for the stress tensor and the heat flux vector we obtain

$$P_{\alpha\beta}^s(1)=m_s\iiint(\mathbf{v}-\mathbf{u}_s)^{\alpha}(\mathbf{v}-\mathbf{u}_s)^{\beta}f_s{}^{(1)}\,d\mathbf{v}=-\mu_s\left(\frac{\partial u_s^{\alpha}}{\partial r_{\beta}}-\frac{1}{2\rho_s}\sum_{\tau\neq s}D_{s\tau}^{\alpha\beta}\right) \tag{15}$$
$$Q_{\alpha}{}^s(1)=\frac{m_s}{2}\iiint(\mathbf{v}-\mathbf{u}_s)^{\alpha}(\mathbf{v}-\mathbf{u}_s)^2f_s{}^{(1)}\,d\mathbf{v}=-\lambda_s\left(\frac{\partial T_s}{\partial r_{\alpha}}+\frac{T_s}{\rho_s}\sqrt{\frac{m_s}{2kT_s}}\sum_{\tau\neq s}H_{s\tau}^{\alpha}\right)$$

where the coefficients of viscosity and thermal conductivity are defined with the use of (14) by

$$\mu_s=-\frac{1}{2}n_skT_sB_s=\frac{5}{2}\frac{kT_s}{\Omega_{ss}^{(2.2)}},\quad\lambda_s=-C_s\frac{5}{4}n_s\sqrt{\frac{2kT_s}{m_s}}$$

Formula (15) shows that in this method stresses and the heat flux vector are affected by diffusion rates and temperatures even in the first approximation. Analysis of formula (15) shows that the terms $D_{s\tau}{}^{\alpha\beta}$ and $H_{s\tau}{}^{\alpha}$ by increasing friction and the heat flux accelerate the gas mixture tendency to equilibrium.

The third method. In many practically important problems the mean velocity of the gas mixture as a whole is low or equal zero, while intensive mixing and heat exchange processes take place in it at the same time. These processes are effected by diffusion rates. For problems of this class the fundamental assumptions of the second method are to be used with the addition of condition

$$nV = n_1u_1 + \ldots + n_Mu_M = 0$$

The set of gasdynamic parameters of the mixture is then evidently defined in the zero approximation by the system of Eqs. (8) in which $W_\alpha{}^s$ and $W_\alpha{}^\tau$ are to be substituted for $u_s{}^\alpha$ and $u_\tau{}^\alpha$.

From this system of zero approximation equations and certain simplifying assumptions about low diffusion rates and small temperature differences we obtain the known formulas (9) of the kinetic theory of gases.

We have thus established the laws of dynamics and heat exchange for gas mixtures and shown that these are much more complex than those for one-component gases. The former are no longer defined by equations of conventional aerohydrodynamics. Intensive interaction takes place between the individual components of gas.

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